

# An Automated Method of Tuning an Attitude Estimator

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## Abstract

Attitude determination is a major element of the operation and maintenance of a spacecraft. There are several existing methods of determining the attitude of a spacecraft. One of the most commonly used methods utilizes the Kalman filter to estimate the attitude of the spacecraft.

Given an accurate model of a system and adequate observations, a Kalman filter can produce accurate estimates of the attitude. If the system model, filter parameters, or observations are inaccurate, the attitude estimates may be degraded. Therefore, it is advantageous to develop a method of automatically tuning the Kalman filter to produce the accurate estimates.

In this paper, a three-axis attitude determination Kalman filter, which uses only magnetometer measurements, is developed and tested using real data. The appropriate filter parameters are found via the Process Noise Covariance Estimator (PNCE). The PNCE provides an optimal criterion for determining the best filter parameters.

## Introduction

The development of light-weight, low-cost spacecrafts that can accomplish complex tasks is essential to the success of many NASA missions (such as Mission to Planet Earth) as well as the success of many commercial missions. One way to ensure a light-weight, low-cost spacecraft is to place constraints on the amount of computer hardware. This constraint demands the use of computationally efficient algorithms that do not require a significant amount of CPU. Consequently, reducing the amount of required hardware improves the performance of the spacecraft and increases the probability of a success.

One of the many functions of a satellite is the gathering and processing of information. In most cases, this information is transmitted to a specified

location. To successfully complete this objective, the orientation of the spacecraft must be known and controlled very precisely. In the past decade, there has been a significant amount of work in the area of attitude determination and attitude control [1-5]. During this period of time, attitude determination algorithms that utilize a combination of the measurements and a mathematical model to estimate the orientation of the spacecraft [6-7] were the most popular. One of the most commonly used and most robust estimators in attitude determination is the Kalman filter. The complexity of this estimator ranges from attitude-only estimator using a QUEST model to an extended Kalman filter with 36 states [8].

Attitude estimators like the Kalman filter are more robust than single-frame methods, such as TRIAD [2], QUEST [4], and FOAM [3]. For example, during periods of near coalignment (the pitch angle is nearly unobservable) or during an eclipse, a sequential estimator, such as the Kalman filter, can provide state estimates by propagating the states with the nominal model. Single-frame methods that rely on measurements can only produce anomalous estimates of the attitude. These estimates may endanger the success of the mission.

The most difficult filter parameter to determine in the Kalman filter is the process noise covariance,  $Q$ . In theory, the process noise is defined as a gaussian process. In real-world applications, the model error can be stochastic, deterministic, or a combination of both. Since the attitude determination problem is very nonlinear, there is a larger possibility for errors in the system model. These errors, along with any stochastic errors, are referred to as modeling errors. As the percentage of non-gaussian modeling errors increases, so does the difficulty in determining an appropriate process noise covariance. Therefore, it is beneficial to develop an algorithm that produces the filter parameters which yield accurate state estimates. In this paper, the PNCE, an algorithm that determines the appropriate filter parameters, is applied to attitude determination. This method provides an automated method of tuning the estimator to obtain reasonable state

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estimates without prior knowledge of the process noise covariance. The PNCE allows for the implementation of Kalman filter type algorithms in real-world applications where the true or the appropriate process noise covariance is not known.

If a spacecraft has rate sensing capability, then the attitude estimation is generally improved over non-rate sensing capable spacecraft. When this capability is not available, the attitude estimation can be improved by estimating the rates based on a model of the spacecraft rotational dynamics. The Solar Anomalous and Magnetospheric Particle Explorer (SAMPEX) [9] and Earth Radiation Budget Satellite (ERBS) [10] are two such spacecraft that do not have rate sensing capabilities. In the case of SAMPEX and ERBS, accurate attitude estimates are ensured by estimating the rates that are based on simple rotational dynamic models along with the attitude. These rotational models improve the overall estimation of the attitude. However, there is no general model for rotational dynamics.

In 1990, Chu and Harvey showed that models of the rotational dynamics could be identified [10-11] and that these models improved the overall estimation of the attitudes. However, obtaining these models can be time-consuming, and the models are only valid for the identified orbit. In 1993, Mook [12-13] described a numerical procedure of finding the appropriate dynamic model of the rates. This procedure can produce models that are valid over a duration longer than the orbit used in the identification. Consequently, this method can be used in prediction. This method is new and has not been applied to many spacecraft. Hence, there is still a need for a simple general model of the rotational dynamics.

To circumvent this problem of not having an accurate dynamic model, a commonly used gyro bias model, based on a Markov process, is used in place of complicated, difficult to obtain rotational dynamic models. This type of simple bias model has been successfully used in the Real-Time Sequential Filter (RTSF) [9]. RTSF uses the gyro bias model along with the basic theory of attitude determination to produce accurate attitude estimates. The accuracy of the estimates from RTSF are dependent on certain filter parameters. In many applications, the RTSF may require a manual tuning. The complexity of this task is a function of the known and unknown dynamics of a spacecraft.

The rest of this paper is divided into three parts Theory, Results, and Conclusion. The

theory section reviews the formulation of the attitude estimator and the PNCE. The result section starts with a definition of the problem and the given filter parameters. Next, these parameters are used along with the PNCE to obtain accurate attitude estimates. The conclusion section summarizes the results and states a few observations.

## Theory

With few exceptions, the dynamics of a spacecraft can be described in terms of classic mechanics. The dynamics of a spacecraft are a function of its orbit and attitude. In this work, only the dynamics associated with the attitude are addressed. The first step in this analysis is the definition of the attitude.

### Attitude Determination: Definition

The attitude of a spacecraft is defined as its orientation. Attitude determination is the process of computing the orientation of the spacecraft relative to either an inertial reference or some object of interest, such as the earth. The attitude determination problem can be stated as: "Given measurements of angles or changes in angles with respect to the spacecraft and a reference, determine the orientation of the spacecraft."

Attitude measurements are produced by sensor such as Fine Sun Sensors (FSS), Three Axis Magnetometers (TAM) sensor, Horizon sensors, Star Trackers, etc. FSS and TAM measurements are used by algorithms like TRIAD [2], QUEST [4], FOAM [3], and the Kalman filter [5,14] to determine the orientation of the spacecraft. The accuracy of the attitude is a function of the sensors and the attitude determination algorithm. Attitude estimators use a combination of several attitude sensor measurements, which are usually associated with the three-axis attitude, to improve the reliability and accuracy of the algorithm.

Three-axis attitude is most conveniently thought of as a coordinate transformation from a reference axis in inertial space to an axis on the spacecraft. For a rigid body, or assumed rigid body spacecraft, the direction of cosine matrix or attitude matrix,  $A$ , represents the coordinate transformation that maps vectors from the reference frame to the body frame. This transformation can be described as

$$e_{body} = Ae_{ref} \quad (1)$$

where  $e_{body}$  and  $e_{ref}$  have components resolved along the body and reference axes, respectively. The attitude matrix consists of three orthogonal, right-handed triads  $\hat{u}$ ,  $\hat{v}$ ,  $\hat{w}$  unit vectors fixed in the body, such that

$$\hat{u} \times \hat{v} = \hat{w} \quad (2)$$

Hence, if one can specify the components of  $\hat{u}$ ,  $\hat{v}$ , and  $\hat{w}$  along the three axes of the coordinate frame, then the orientation can be determined completely.

The attitude matrix is a real orthogonal matrix that has many different orientation parameterizations. The type of parameterization used is dependent on the application. A commonly used parameterization is the Euler parameterization (Euler angles). One of the benefits of using this type of parameterization is that the Euler angles have some physical significance. Another type of parameterizations is the quaternions parameterization, which is also known as the Euler symmetric parameterization.

### Quaternion Parameterization

The term quaternion, which is sometimes referred to as Euler symmetric parameters, was first used by Hamilton [15] in 1843.

Many authors [16-20] have discussed the use of this four-parameter representation of the attitude. The advantage of using quaternions over Euler angles is that quaternions are not singular, unlike Euler angles. Because of its advantage, today, most attitude estimators utilize quaternion attitude representation instead of Euler angles. Quaternions are also easier to work with. However, the quaternions representation is not unique. This characteristic is discussed later in the text. The quaternions are defined by three primary parameters and an auxiliary parameter

$$\begin{aligned} [q_1 \quad q_2 \quad q_3] &= \bar{e} \sin\left(\frac{\Phi}{2}\right) \\ q_4 &= \cos\left(\frac{\Phi}{2}\right) \end{aligned} \quad (3)$$

where:

$\bar{e}$  is a unit vector corresponding to the axis of rotation

$\Phi$  is the angle of rotation

The quaternion parameterization is nonsingular because the quaternions are not independent. The quaternions are related by the following normalization constraint

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \quad (4)$$

Quaternions can be defined in terms of the attitude matrix or the Euler angles. The reverse is also true, that the attitude matrix can be expressed in terms of the quaternions

$$A(q) = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \quad (5)$$

$$A(q) = (q_4^2 - \mathbf{q}^2)1 + 2\mathbf{q}\mathbf{q}^T - 2q_4\mathcal{Q} \quad (6)$$

Being able to represent the attitude matrix as an algebraic function of the quaternions is another computational advantage of the quaternion representation. Now that the quaternions representation and the attitude matrix have been defined, the kinematics of the orientations and dynamic equations of motion can be addressed.

### Kinematics and Dynamic equations of motion

Kinematics is the study of the orientation of the object rotating (with its body axis fixed on the body of the object) relative to some global frame of reference, which results in equations of motion of the orientation. These equations of motion are independent of the forces associated with the particular problem.

As defined in the literature, the kinematics relation for the orientation is

$$\dot{q} = \frac{1}{2}\Omega(w)q \quad (7)$$

where the expression  $\Omega$  of a variable  $\alpha$  can be represented as

$$\Omega(\alpha) = \begin{bmatrix} -[\alpha \times] & \alpha \\ -\alpha^T & 0 \end{bmatrix}$$

$$[\alpha \times] = \begin{bmatrix} 0 & -\alpha_3 & \alpha_2 \\ \alpha_3 & 0 & -\alpha_1 \\ -\alpha_2 & \alpha_1 & 0 \end{bmatrix}$$

If  $\alpha$  is defined as

$$\bar{\alpha} = [\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad 0]^T$$

then  $\Omega(\alpha)\theta = \theta \otimes \alpha$ , where quaternion multiplication,  $q_{v,1} \otimes q_{v,2}$ , is defined as

$$q_{v,1} \otimes q_{v,2} = \begin{bmatrix} r_1 \\ p_1 \end{bmatrix} \otimes \begin{bmatrix} r_2 \\ p_2 \end{bmatrix} = \begin{bmatrix} r_1 \times r_2 + p_1 r_2 + p_2 r_1 \\ p_1 p_2 - r_1 \cdot r_2 \end{bmatrix} \quad (8)$$

The attitude dynamic equations of motion, relating the time derivative of the angular momentum and the applied torque, is

$$\frac{dL}{dt} = N - w \times L = I \frac{dw}{dt} \quad (9)$$

where  $N$  is the torque vector

$$N = \sum_{i=1}^n r_i \times F_i \quad (10)$$

$F$  is an external force. A spacecraft equipped with reaction or momentum wheels is not considered a rigid body. Therefore, the attitude dynamics equation must be modified

$$\frac{dL}{dt} = N - [I^{-1}(L - h)] \times L = I \frac{dw}{dt} \quad (11)$$

The body angular rates associated with this system are defined as

$$w = I^{-1}(L - h) \quad (12)$$

The difference between the true quaternion and the estimated quaternion is

$$\hat{q} = q_{true} \otimes \delta q \quad (13)$$

where  $\hat{q}$  is the estimated quaternion and  $\delta q$  is the difference between the estimated and actual quaternion. Substituting this into the dynamic equation for the estimate (7) yields

$$\begin{aligned} \frac{d}{dt}(q_{true} \otimes \delta q) &= \frac{1}{2} \Omega(\hat{w})(q_{true} \otimes \delta q) \\ \frac{1}{2} q_{true} \otimes w_{true} \otimes \delta q + q_{true} \otimes \frac{d\delta q}{dt} &= \frac{1}{2} q_{true} \otimes \delta q \otimes \hat{w} \\ 2 \frac{d\delta q}{dt} &= \delta q \otimes \hat{w} - w_{true} \otimes \delta q \end{aligned} \quad (14)$$

Note,  $\delta q$  is unique because it is defined as

$$\delta q = [\alpha \quad 1]^T \quad (15)$$

### Propagation equations

In this section, the estimation algorithm is formulated using the same filter formulation presented as Mook [12-13]. This formulation is mathematically rigorous and produces accurate estimates.

The propagation equations are based on the equations of motion. The seventh order state vector for this filter is

$$x(t) = \begin{bmatrix} \bar{q}(t) \\ L(t) \end{bmatrix} \quad (16)$$

The dynamic equations are

$$\dot{q} = \frac{1}{2} \Omega(\hat{w})q \quad (17)$$

$$\frac{dL}{dt} = N - [I^{-1}(L - h)] \times L = I \frac{d\hat{w}}{dt} \quad (18)$$

where the body angular are

$$\hat{w} = I^{-1}(L - h) \quad (19)$$

$N$  is defined by equation (10). The state space representation is

$$\frac{dX}{dt} = \begin{bmatrix} \frac{1}{2} \Omega(\hat{w}) & 0_{4,3} \\ 0_{3,4} & [\hat{w} \times] \end{bmatrix} \hat{X} + \begin{bmatrix} 0_{4,1} \\ I_{3,3} \end{bmatrix} N \quad (20)$$

$$\frac{dX}{dt} \equiv f(w)X + BN \quad (21)$$

For nonlinear systems, the error analysis is based on a linearization of the system. Defining

$F \equiv \frac{\partial f}{\partial X}$ , the error covariance can be written as

$$\frac{dP}{dt} = FP + PF^T + Q \quad (22)$$

### Update equations

The update equations for this filter formulation are the same as in the RTSF [21] formulation

$$y = [\hat{V}_b \times] \alpha(-) + \Delta V_b \quad (23)$$

The sensitivity matrix  $H$  can be defined as

$$H = [[\hat{V}_b \times] \quad 0_{3,3}] \quad (24)$$

Consequently,  $y$  is linearly related to the state error

$$y = Hx(-) + \Delta V_b \quad (25)$$

The update equations are

$$\begin{bmatrix} \alpha^* \\ \delta L \end{bmatrix} = \Delta x = Ky \quad (26)$$

$$\delta q = \begin{bmatrix} \alpha^* \\ 1 \end{bmatrix} \quad (27)$$

$$\dot{q}^* = \dot{q}^- \otimes \delta q \quad (28)$$

$$\dot{L}^* = \dot{L}^- + \delta L$$

$$P^* = [I - KH]P^- \quad (29)$$

$$K = P^- H^T [H P^- H^T + R]^{-1} \quad (30)$$

### Summary of algorithm

To summarize this algorithm, consider the following steps taken during the execution of the filter. It is assumed that all filter parameters are known ahead of time.

Given

- The initial attitude quaternion  $\hat{q}_k(+)$
  - The initial rate error  $\hat{L}_k(+)$
  - The initial error covariance  $P_k(+)$
1. Propagate the states and error covariance using the updated or initial values of the state and the error. (17) and (18)
  2. Compute the residual. (23)
  3. Compute the update state, update covariance and Kalman gain. (26-30)
  4. Go to 1

In the filter formulation above, the process noise is assumed to be a known gaussian process. For real-world application, the process noise is not known exactly. Therefore, the next logical step is to devise an algorithm that produces the appropriate covariance to produce accurate state estimates. The method used in this paper is referred to as the PNCE .

### PNCE

The PNCE [21] is a parameter optimization technique that identifies filter parameters that produce near-optimal state estimates in the presence of model error. This algorithm can be thought of as an external optimality criterion for obtaining filter parameters, in particular the process noise covariance,  $Q$ . In the formulation presented here, the process noise covariance matrix is assumed diagonal. This diagonal form simplifies the optimization and is frequently used in research and applications. The accuracy of the PNCE algorithm is a function of the optimization process and the complexity of the functional form of process noise covariance.

Figure 1 contains a flow chart of the PNCE algorithm. The flow chart describes the steps taken by the PNCE to solve for the appropriate

covariance matrix. The major steps of the PNCE are given below:

- 1) Use  $Q_i$  in the Kalman-type filter to calculate the state estimates.
  - a) For the initial step,  $Q_i$  is an initial covariance provided by the user.
- 2) The state estimates are used to evaluate the costs and constraints in the cost/constraint routine.
- 3) If the cost is not minimized or the constraints not satisfied, then the optimization routine calculates a new  $Q_i$  and return to step 1. If the costs are minimized and the constraints are satisfied, then the appropriate process noise covariance is found and PNCE stops.

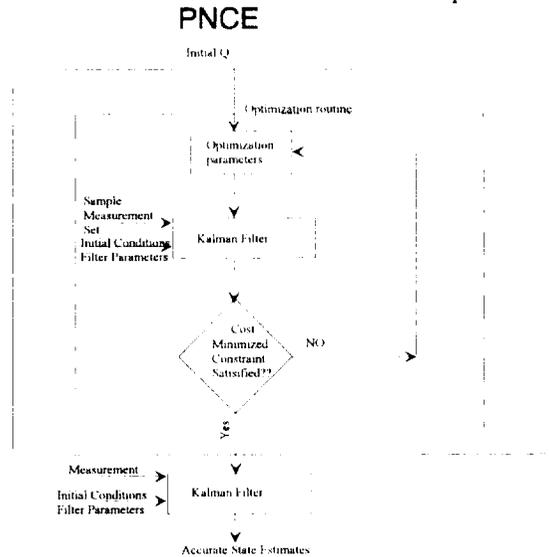


Figure 1 Flow chart of the PNCE algorithm

There are several advantages to this algorithm. First, it provides a consistent method of determining the appropriate process noise covariance. Another advantage is that the physical model error does not have to be a gaussian process to obtain accurate results. The physical model error is the model error associated with real-world applications. This error is not confined to gaussian process as defined in the original Kalman filter formulation. This allows the filter to be implemented in non-ideal environments, such as in real-world applications.

As shown in Figure 1, the PNCE is made up of several different components. The most important of these components is the cost/constraint routine.

### Cost/Constraint Routine

In this section, the cost/constraint component of the PNCE is discussed. The cost/constraint routine is the second component of the PNCE algorithm. This component defines the accuracy of the estimate of the filter parameter estimation. This component is user and problem dependent.

### Covariance Constraint

A major part of the cost/constraint routine is the Covariance Constraint. The covariance constraint was formulated by Mook and Junkins in 1985 [23]. This concept was developed as a part of another estimation algorithm, the Minimum Model Error algorithm. The covariance constraint states that the measurement-minus-estimate error covariance must match the measurement-minus-truth error covariance if the estimates mirror the truth. When this occurs the covariance constraint is satisfied. In the PNCE, the covariance constraint is a function of the process noise covariance,  $Q$ . The correct  $Q$  should produce estimates that fit the actual measurements with approximately the same error covariance as the actual measurement fit the truth. Therefore, the measurement noise distribution does not have to be completely gaussian to obtain accurate estimates. The covariance constraint can be expressed mathematically as:

$$E[(\tilde{y} - \hat{y})^T (\tilde{y} - \hat{y})] = R \quad (31)$$

where:

$R$  ( $m \times m$ ) is the measurement noise covariance

$\tilde{y}$  ( $m \times 1$ ) is the measurement vector

$\hat{y}(t)$  is the output estimate vector

The covariance constraint is the primary cost function used by the PNCE. However, other costs functions and constraints can be utilized to improve the results of the parameter identification. These additional functions and constraints, if used, are dependent on the application.

## Simulation Results

In this section, the PNCE algorithm is used to develop an accurate attitude determination estimator based on real data. This data is obtained from telemetry files provided by NASA Goddard Space Flight Center, Flight Dynamics Branch. These telemetry files contained a nominal pass (nonevent) data set. A nonevent data set is used to ensure that the "truth" (from TRIAD) is available to evaluate the performance of the filter.

To maintain consistency, the same numerical values of the filter parameter used in the RTSF report [27] are used here. The inertia matrix,  $I$ , and the wheel inertial,  $I_{whl}$ , are

$$I = \begin{bmatrix} 15.516 & 0.0 & 0.0 \\ 0.0 & 21.621 & -0.194 \\ 0.0 & -0.194 & 15.234 \end{bmatrix} \text{ kg} - m^2$$

$$I_{whl} = 0.0041488 \text{ kg} - m^2$$

The total torque vector,  $N$ , and the angular momentum,  $h$ , are known inputs to the system. In this simulation study, the measurement noise covariance is obtained from the SAMPEX evaluation report [21].

For the Fine Sun Sensors (FSS) measurements,  $\sigma_{FSS}^2 = 6.346 \times 10^{-6}$ . The error in the FSS measurement is primarily due to the digitization noise (0.5 deg). For the TAM measurements, the digitization noise is only about 0.3  $mG$  and  $\sigma_{TAM}^2 = 3mG$ . The time constant used in the gyro bias model is  $\tau = 5.0$ sec (for playback). A distinctive feature of telemetry SAMPEX data is the large amount of white noise associated with the torques. The magnitude of the torques associated with this noise is  $10^{-2}$ , which far exceeds the magnitude of the environmental torques of  $10^{-6}$ .

The noise statistics, along with physical insight, are used to determine the growth rate of the error covariance. The growth rate is

$$(3 \times 10^{-3}) \Delta t \frac{\text{rad}^2}{\text{sec}^3}$$

This is an approximation of the process noise covariance,  $Q$ . Using this approximation, physical insight and tuning, the appropriate  $Q$  can be found, but this process can be time-consuming. In this experiment, an automated method of tuning the estimator, the PNCE, is used to determine the appropriate filter parameter.

Since the attitude estimator developed here only requires magnetometer data, some of the accuracy and reliability may be lost. This simulation is used to demonstrate that an accurate estimator can be developed automatically. To ensure robustness in the presence of additional modeling errors, the initial conditions are perturbed from their correct values.

For this study, the process noise covariance is assumed to be of the following form

$$Q = \begin{bmatrix} I_{3 \times 3} q_q & 0 \\ 0 & I_{3 \times 3} q_l \end{bmatrix}$$

where  $q_q$  and  $q_l$  are to be determined. Using the measurement noise, the PNCE determines the appropriate values for  $q_q$  and  $q_l$ .

$$q_q = 1.e - 2 \qquad q_l = 9.64e - 8$$

During non-event passes, good data from both FSS and TAM, TRIAD is considered to be near-perfect. Therefore, TRIAD is considered to be the Truth. A nonevent pass is part of an orbit or the whole orbit where an eclipse or other anomalies do not occur.

Figure 2-4 contain plots of the Roll, Pitch, and Yaw of the truth and of the estimator using the standard formulation. The state estimates are initially off but then converge to the truth quickly. This initial error is due to the initial condition error. Figure 5 contains a plot of the output estimates and the TAM measurement.

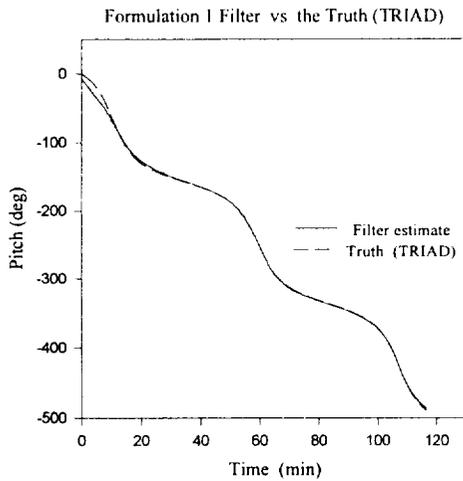


Figure 2 The estimated and true Pitch

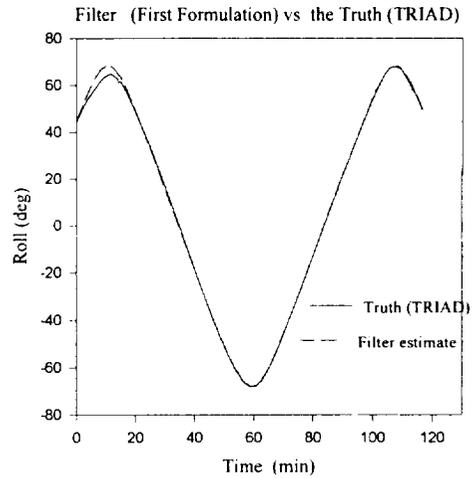


Figure 3 The estimated and true Roll

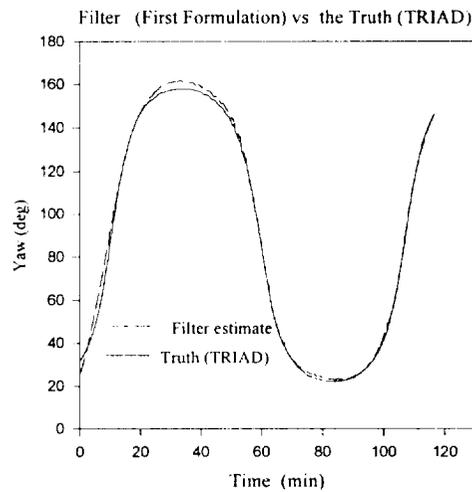
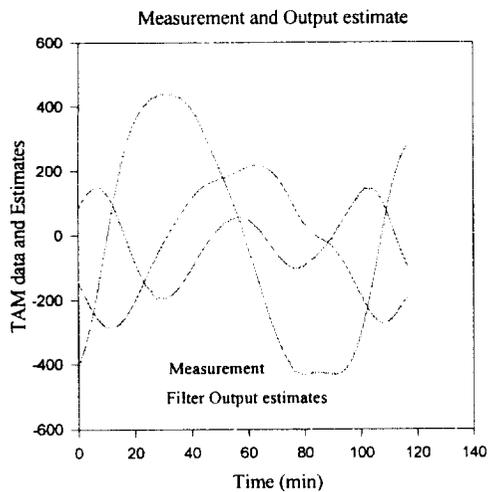


Figure 4. The estimated and true Yaw



**Figure 5** The TAM measurements and the output estimates

In the plots above, filter produce accurate estimates of the attitude and the output (the TAM measurements). Even though the initial conditions are perturbed, the filter is able to converge to the truth quickly. This illustrates the robustness of the PNCE and the present filter formulation.

## Conclusion

The purpose of this paper is to demonstrate a new method for obtaining accurate state estimates for a three-axis magnetometer attitude estimator. This method, the PNCE, used statistical properties and a data set to determine the appropriate process noise covariance. The PNCE algorithm is utilized to develop an attitude filter. This filter formulations produced accurate attitude and output estimates.

From the results in this paper, it has been shown that the PNCE estimator is a robust algorithm that can account for deterministic linear model uncertainty and error in the initial conditions or the filter parameters.

## References

- [1] Wahba, G., "A Least-Squares Estimate of Satellite Attitude," *SIAM Review*, Vol. 7, No. 3, July 1965, pp. 409.
- [2] Lerner, G. M., "Three-Axis Attitude Determination," *Spacecraft Attitude Determination and Control*, edited by J. R. Wertz, D. Reidel Publishing Co., Dordrecht, The Netherlands, 1978, pp. 420-428.
- [3] Markley F., L., "Attitude Determination from Vector Observations: A Fast Optimal Matrix Algorithm," *Journal of the Astronautical Sciences*, Vol. 41, No. 2, April-June, 1993, pp. 261-280.
- [4] Shuster, M. D., Oh, S. D., "Three-Axis Attitude Determination from Vector Observation," *Journal of Guidance and Control*, Vol. 4, No. 1, 1981, pp. 70-77.
- [5] Shuster, M., "Kalman Filtering of Spacecraft Attitude and the Quest Model," *Journal of the Astronautical Sciences*, Vol. 38, No. 3, July-September, 1990, pp. 377-393.
- [6] Martel F., Pal, P. K., and Psiaki M. L., "Three-Axis Attitude Determination via Kalman Filtering of Magnetometer Data," *Proceedings of the 1988 Flight Mechanics Estimation Theory Symposium*, NASA Goddard, 1988.
- [7] Heyler, G. A., "Attitude Determination by Enhanced Kalman Filtering Using Euler Parameter Dynamics and Rotational Update Equations," *Proceedings of the AAS/AIAA Astrodynamics Specialist Conference*, No. A81-45832., Nevada, Aug. 1981.
- [8] Deutschmann, J., Rokni, M., and Bar-Itzhack, I., "Comparison and Testing of Extended Kalman Filters for Attitude Estimation of the Earth Radiation Budget Satellite," *Proceedings of the Flight Mechanics/Estimation Theory Symposium*, NASA Goddard Space Flight Center, 1990.
- [9] Challa, M. S., Natanson, G. A., Baker, D. E., and Deutschmann, J. K., "Advantages of Estimating Rate Corrections During Dynamic Propagation of Spacecraft Corrections During Dynamic Propagation of Spacecraft Rates-Applications to Real-Time Attitude Determination of SAMPEX," *Proceedings of the Flight Mechanics/Estimation Theory Symposium*, NASA Goddard, 1994, pp. 481-495.
- [10] Chu, D., Harvie, E., "Accuracy of the ERBS Definitive Attitude Determination System in the Presence of Propagation Noise," *Proceedings of the Flight Mechanics/Estimation Theory Symposium*, NASA Goddard, 1990, pp. 97-114.
- [11] Chu, D., Glickman, J., and Harvie, E., "Improvements in ERBS Attitude Determination Without Gyros," *Proceedings of the Flight*

- Mechanics/Estimation Theory Symposium*, NASA Goddard, 1992, pp. 185-200.
- [12] Crassidis, J., Mook, D. J., De Peña, J., and Meyer, T., "Robust Attitude and Attitude Rate Estimation for Sampex," *Proceedings of the AAS/AIAA Astrodynamics Specialist Conference*, Cocoa Beach, Florida, February, 1994.
- [13] De Peña, J., Crassidis, J., Mook, D. J., and Meyer, T., "MME-Based Attitude Dynamics Identification and Estimation for Sampex," *Proceedings of the Flight Mechanics/Estimation Theory Symposium Conference*, NASA Goddard, Greenbelt, Maryland, May, 1994.
- [14] Shuster, M. D., "The quaternion in the Kalman filter," *Proceedings of the AAS/AIAA Astrodynamics Specialist Conference*, Victoria, B.C., Canada, Aug. 16-19, 1993.
- [15] Hamilton, W. R., *Elements of Quaternions*, Longmans, Green Co., London 1866.
- [16] Markley F., L., "Attitude Determination and Parameter Estimation Using Vector Observation: Theory," *Journal of the Astronautical Sciences*, Vol. 37, No. 1, January-March, 1989, pp. 41-58.
- [17] Stuelpnagel, J., "On the Parameterization of the Three-Dimension Rotational Group," *SIAM Review*, Vol. 6, No. 1, Oct. 1964, pp. 422-430.
- [18] Wilcox, J. C., "A New Algorithm for Strapped-Down Inertial Navigation," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-3, Sept. 1967, pp. 796-802.
- [19] Markley F., L., "Parameterizations of the Attitude," edited by J.R. Wertz, D. Reidel, Dordrecht, the Netherlands, 1978.
- [20] Wertz, J. R., *Spacecraft Attitude Determination and Control*, Kluwer, 1991.
- [21] Challa, M. S., "Solar, Anomalous, Magnetospheric Particle Explorer (SAMPEX) Real-Time Sequence Filter (RTSF)," *Evaluation REPORT*, NASA Goddard, 1993.
- [22] Mason, P. A. C., and Mook, D.J., "A Process Noise Covariance Estimator," *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, Scottsdale, AZ, August, 1994.
- [23] Mook, D. J., and Junkins, J. L., "Minimum Model Error Estimation for Poorly Modeled Dynamic Systems," *AIAA Journal*

*of Guidance, Control and Dynamics*, Vol. II, No. 3, May-June 1988, pp. 256-261.

